ALGEBRA and TRIGONOMETRY

Real Mathematics, Real People

7e

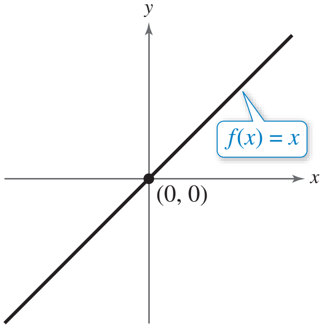
Solutions, Interactivity, Videos, & Tutorial Help at LarsonPrecalculus.com

Ron Larson

Library of Parent Functions Summary

Linear Function ***(p. 87)***

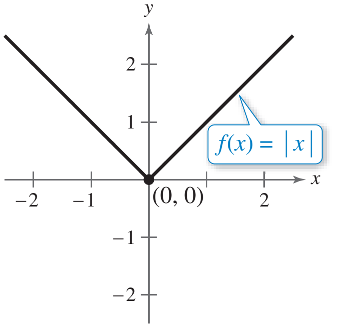
$f(x) = x$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing

Absolute Value Function ***(p. 100)***

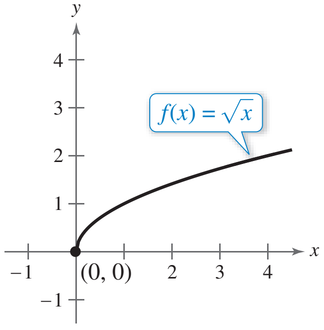
$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
y-axis symmetry

Square Root Function ***(p. 101)***

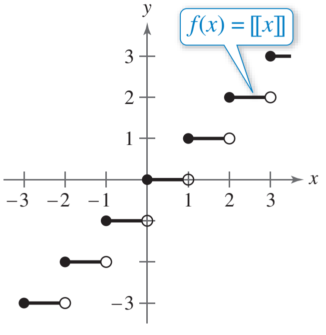
$f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$

Greatest Integer Function ***(p. 115)***

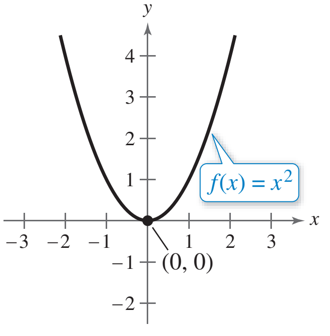
$f(x) = [\![x]\!]$



Domain: $(-\infty, \infty)$
Range: the set of integers
x-intercepts: in the interval $[0, 1)$
y-intercept: $(0, 0)$
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value

Quadratic Function ***(p. 246)***

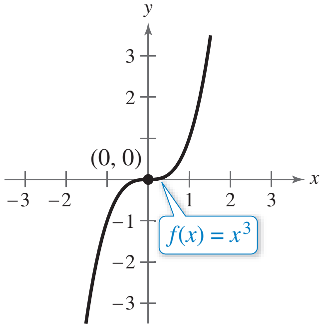
$f(x) = x^2$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
Axis of symmetry: $x = 0$
Relative minimum or vertex: $(0, 0)$

Cubic Function ***(p. 255)***

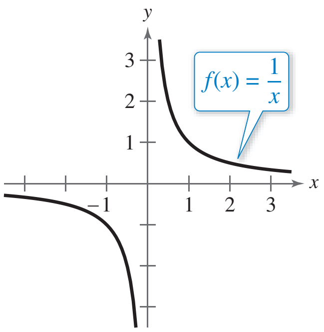
$f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing on $(-\infty, \infty)$
Odd function
Origin symmetry

Rational Function ***(p. 298)***

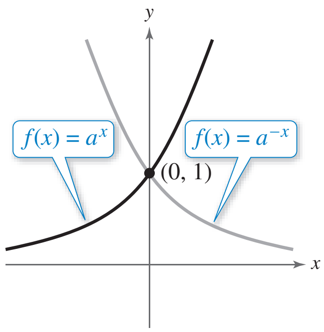
$f(x) = \dfrac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
No intercepts
Decreasing on $(-\infty, 0)$ and $(0, \infty)$
Odd function
Origin symmetry
Vertical asymptote: y-axis
Horizontal asymptote: x-axis

Exponential Function ***(p. 326)***

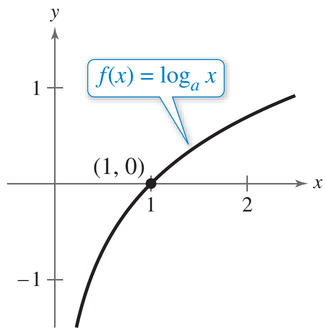
$f(x) = a^x, a > 1$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Intercept: $(0, 1)$
Increasing on $(-\infty, \infty)$
for $f(x) = a^x$
Decreasing on $(-\infty, \infty)$
for $f(x) = a^{-x}$
x-axis is a horizontal asymptote
Continuous

Logarithmic Function ***(p. 339)***

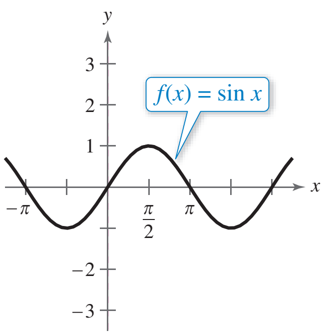
$f(x) = \log_a x, a > 1$



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(1, 0)$
Increasing on $(0, \infty)$
y-axis is a vertical asymptote
Continuous
Reflection of graph of $f(x) = a^x$
in the line $y = x$

Sine Function ***(p. 433)***

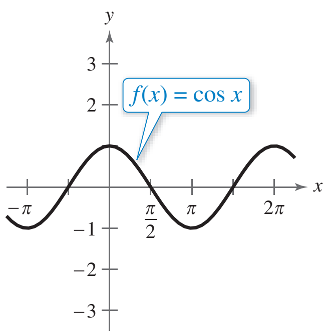
$f(x) = \sin x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Odd function
Origin symmetry

Cosine Function ***(p. 433)***

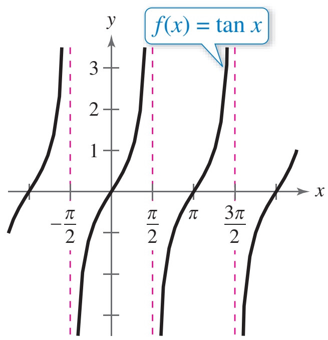
$f(x) = \cos x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $\left(\dfrac{\pi}{2} + n\pi, 0\right)$
y-intercept: $(0, 1)$
Even function
y-axis symmetry

Tangent Function ***(p. 444)***

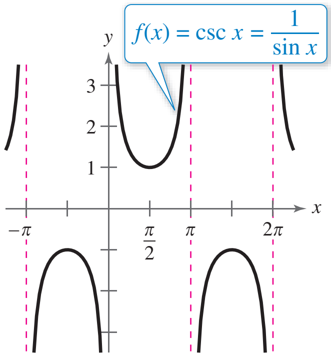
$f(x) = \tan x$



Domain: $x \neq \dfrac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Vertical asymptotes: $x = \dfrac{\pi}{2} + n\pi$
Odd function
Origin symmetry

Cosecant Function ***(p. 447)***

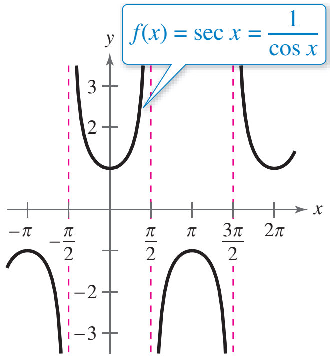
$f(x) = \csc x$



Domain: $x \neq n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
No intercepts
Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Secant Function ***(p. 447)***

$f(x) = \sec x$



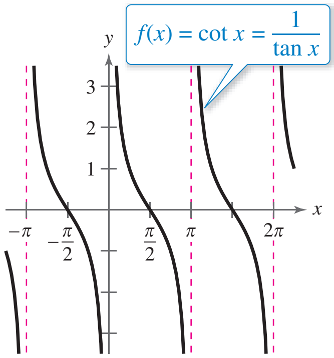
Domain: $x \neq \dfrac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
y-intercept: $(0, 1)$
Vertical asymptotes:

$$x = \frac{\pi}{2} + n\pi$$

Even function
y-axis symmetry

Cotangent Function ***(p. 446)***

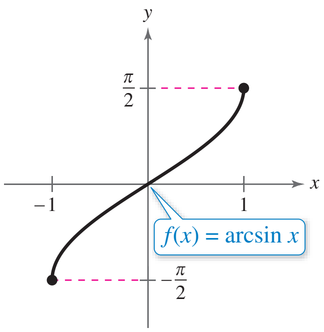
$f(x) = \cot x$



Domain: $x \neq n\pi$
Range: $(-\infty, \infty)$
Period: π
x-intercepts: $\left(\dfrac{\pi}{2} + n\pi, 0\right)$
Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Inverse Sine Function ***(p. 459)***

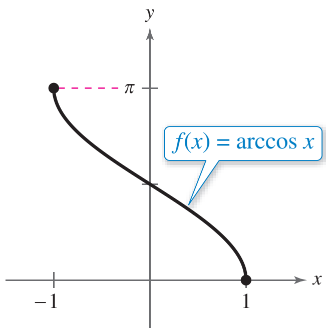
$f(x) = \arcsin x$



Domain: $[-1, 1]$
Range: $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
Intercept: $(0, 0)$
Odd function
Origin symmetry

Inverse Cosine Function ***(p. 459)***

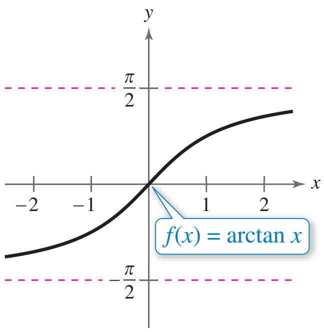
$f(x) = \arccos x$



Domain: $[-1, 1]$
Range: $[0, \pi]$
y-intercept: $\left(0, \dfrac{\pi}{2}\right)$

Inverse Tangent Function ***(p. 459)***

$f(x) = \arctan x$



Domain: $(-\infty, \infty)$
Range: $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$
Intercept: $(0, 0)$
Horizontal asymptotes: $y = \pm\dfrac{\pi}{2}$
Odd function
Origin symmetry

Algebra and Trigonometry

Real Mathematics, Real People

Seventh Edition

Ron Larson
The Pennsylvania State University
The Behrend College

With the assistance of David C. Falvo
The Pennsylvania State University
The Behrend College

CENGAGE Learning®

Australia • Brazil • Mexico • Singapore • United Kingdom • United States

Algebra and Trigonometry: Real Mathematics, Real People
Seventh Edition

Ron Larson

Senior Product Director: Richard Stratton
Product Manager: Gary Whalen
Senior Content Developer: Stacy Green
Associate Content Developer: Samantha Lugtu
Product Assistant: Katharine Werring
Media Developer: Lynh Pham
Senior Marketing Manager: Mark Linton
Content Project Manager: Jill Quinn
Manufacturing Planner: Doug Bertke
IP Analyst: Christina Ciaramella
IP Project Manager: John Sarantakis
Compositor: Larson Texts, Inc.
Text and Cover Designer: Larson Texts, Inc.
Cover Images: kentoh/Shutterstock.com;
mistery/Shutterstock.com; Georgios Kollidas/Shutterstock.com;
PureSolution/Shutterstock.com; mtkang/Shutterstock.com

Library of Congress Control Number: 2014947837

Student Edition
ISBN-13: 978-1-305-07173-5

Cengage Learning
20 Channel Center Street
Boston, MA 02210
USA

Printed in the United States of America

Print Number: 01 Print Year: 2014

Contents

3 ▷ Polynomial and Rational Functions 243

4 ▷ Exponential and Logarithmic Functions 323

5 ▷ Trigonometric Functions 397

Preface

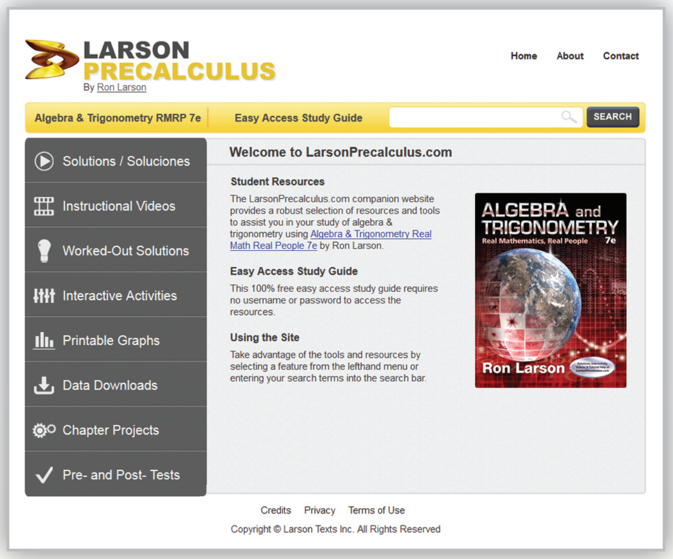
Welcome to *Algebra and Trigonometry: Real Mathematics, Real People*, Seventh Edition. I am proud to present to you this new edition. As with all editions, I have been able to incorporate many useful comments from you, our user. And while much has changed in this revision, you will still find what you expect—a pedagogically sound, mathematically precise, and comprehensive textbook. In this book you will see how algebra and trigonometry are used by real people to solve real-life problems and make real-life decisions.

In addition to providing real and relevant mathematics, I am pleased and excited to offer you something brand new—a companion website at **LarsonPrecalculus.com.** My goal is to provide students with the tools they need to master algebra and trigonometry.

New To This Edition

NEW LarsonPrecalculus.com

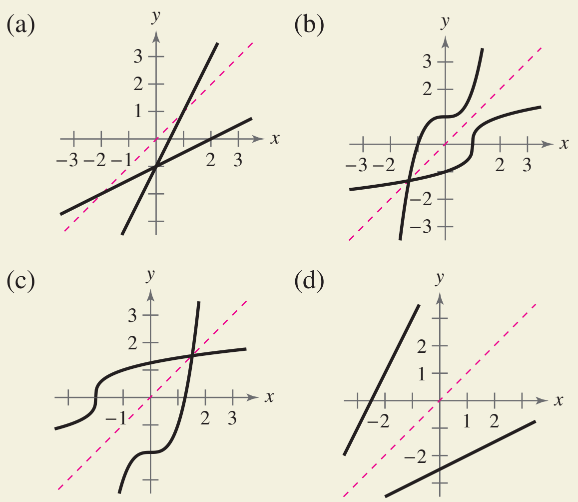
This companion website offers multiple tools and resources to supplement your learning. Access to these features is free. View and listen to worked-out solutions of Checkpoint problems in English or Spanish, explore examples, download data sets, watch lesson videos, and much more.



NEW Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at LarsonPrecalculus.com.

130. **HOW DO YOU SEE IT?** Decide whether the two functions shown in each graph appear to be inverse functions of each other. Explain your reasoning.



NEW How Do You See It?

The How Do You See It? feature in each section presents an exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

NEW Data Spreadsheets

Download these editable spreadsheets from LarsonPrecalculus.com and use the data to solve exercises.

REVISED Exercise Sets

The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant and to include all topics our users have suggested. The exercises have been **reorganized and titled** so you can better see the connections between examples and exercises. Multi-step exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations.

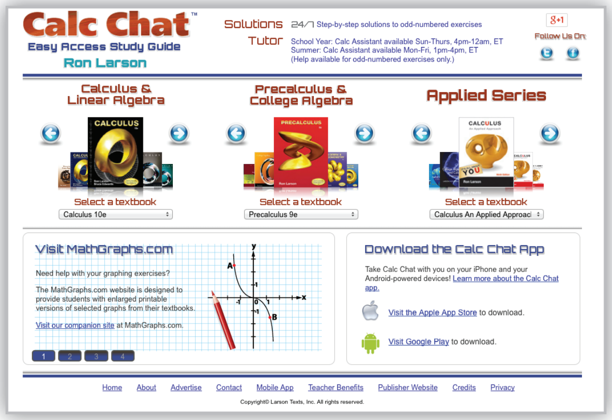
REVISED Remarks

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, address special cases, or show alternative or additional steps to a solution of an example.

Trusted Features

Calc Chat

For the past several years, an independent website—CalcChat.com—has provided free solutions to all odd-numbered problems in the text. Thousands of students have visited the site for practice and help with their homework.



Side-By-Side Examples

Throughout the text, we present solutions to examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses different learning styles.

Why You Should Learn It Exercise

An engaging real-life application of the concepts in the section. This application exercise is typically described in the section opener as a motivator for the section.

Library of Parent Functions

To facilitate familiarity with the basic functions, several elementary and nonelementary functions have been compiled as a Library of Parent Functions. Each function is introduced at its first appearance in the text with a definition and description of basic characteristics. The Library of Parent Functions Examples are identified in the title of the example and there is a Review of Library of Parent Functions after Chapter 4. A summary of functions is presented on the inside cover of this text.

Make a Decision Exercises

The Make a Decision exercises at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These exercises are offered online at LarsonPrecalculus.com.

Chapter Openers

Each Chapter Opener highlights a real-life modeling problem, showing a graph of the data, a section reference, and a short description of the data.

Explore the Concept

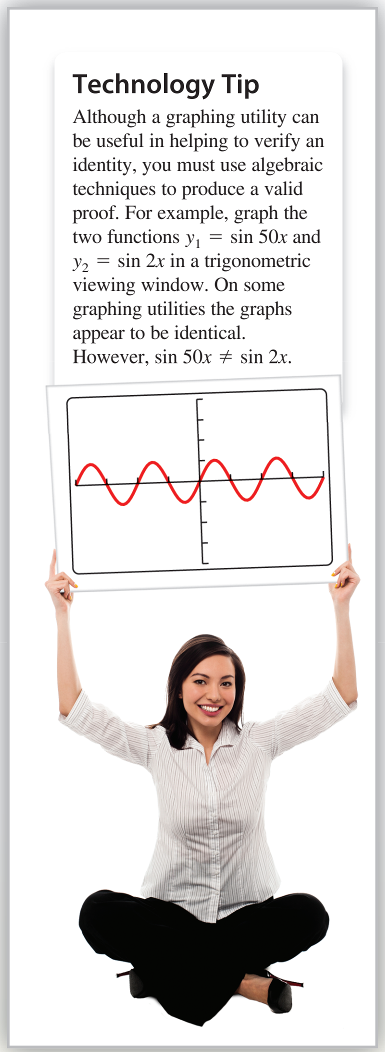
Each Explore the Concept engages you in active discovery of mathematical concepts, strengthens critical thinking skills, and helps build intuition.

Explore the Concept

Complete the following:

$i^1 = i$	$i^7 =$
$i^2 = -1$	$i^8 =$
$i^3 = -i$	$i^9 =$
$i^4 = 1$	$i^{10} =$
$i^5 =$	$i^{11} =$
$i^6 =$	$i^{12} =$

What pattern do you see? Write a brief description of how you would find i raised to any positive integer power.



What's Wrong?

Each What's Wrong? points out common errors made using graphing utilities.

Technology Tip

Technology Tips provide graphing calculator tips or provide alternative methods of solving a problem using a graphing utility.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol $\int$.

Algebraic-Graphical-Numerical Exercises

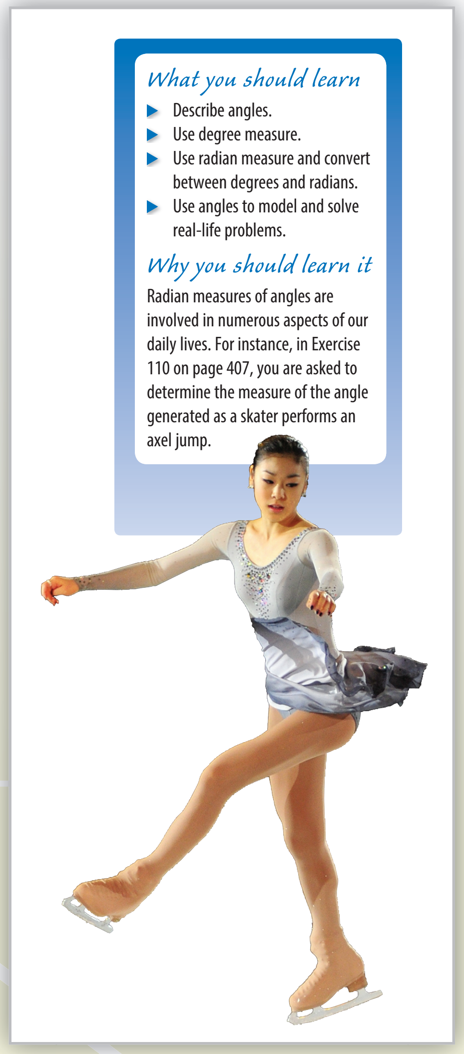
These exercises allow you to solve a problem using multiple approaches—algebraic, graphical, and numerical. This helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result.

Modeling Data Exercises

These multi-part applications that involve real-life data offer you the opportunity to generate and analyze mathematical models.

Vocabulary and Concept Check

The Vocabulary and Concept Check appears at the beginning of the exercise set for each section. Each of these checks asks fill-in-the-blank, matching, and non-computational questions designed to help you learn mathematical terminology and to test basic understanding of that section's concepts.



What you should learn/Why you should learn it

These summarize important topics in the section and why they are important in math and in life.

Chapter Summaries

The Chapter Summary includes explanations and examples of the objectives taught in the chapter.

Error Analysis Exercises

This exercise presents a sample solution that contains a common error which you are asked to identify.

ENHANCED WebAssign

Enhanced WebAssign combines exceptional algebra and trigonometry content with the most powerful online homework solution, WebAssign. Enhanced WebAssign engages you with immediate feedback, rich tutorial content and interactive, fully customizable eBooks (YouBook) helping you to develop a deeper conceptual understanding of the subject matter.

Instructor Resources

Complete Solutions Manual

• ISBN-13: 9781305252530

This manual contains solutions to all exercises from the text, including Chapter Review Exercises and Chapter Tests. This manual is found on the Instructors Companion Site.

Test Bank

• ISBN-13: 9781305252547

This supplement includes test forms for every chapter of the text, and is found on the instructor companion site.

Text-Specific DVDs

• ISBN-13: 9781305252516

These text-specific DVDs cover all sections of the text—providing explanations of key concepts as well as examples, exercises, and applications in a lecture-based format.

Enhanced WebAssign

Printed Access Card: 9781285858333
Instant Access Code: 9781285858319

Enhanced WebAssign combines exceptional mathematics content with the most powerful online homework solution, WebAssign. Enhanced WebAssign engages your students with immediate feedback, rich tutorial content, and an interactive, fully customizable eBook, Cengage YouBook helping students to develop a deeper conceptual understanding of the subject matter.

Instructor Companion Site

Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online via www.cengage.com/login. Access and download PowerPoint presentations, images, instructor's manual, and more.

Cengage Learning Testing Powered by Cognero

• ISBN-13: 9781305259010

CLT is a flexible online system that allows you to author, edit, and manage test bank content; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you want. This is available online via www.cengage.com/login.

Student Resources

Student Solutions Manual

• ISBN-13: 9781305252493

Contains fully worked-out solutions to all of the odd-numbered exercises in the text, giving you a way to check your answers and ensure that you took the correct steps to arrive at an answer.

Enhanced WebAssign

Printed Access Card: 9781285858333
Instant Access Code: 9781285858319

Enhanced WebAssign combines exceptional mathematics content with the most powerful online homework solution, WebAssign. Enhanced WebAssign engages you with immediate feedback, rich tutorial content, and an interactive, fully customizable eBook, Cengage YouBook helping you to develop a deeper conceptual understanding of the subject matter.

CengageBrain.com

To access additional course materials, please visit www.cengagebrain.com. At the CengageBrain.com home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where these resources can be found.

Acknowledgments

I would like to thank my colleagues who have helped me develop this program. Their encouragement, criticisms, and suggestions have been invaluable to me.

Reviewers

Hugh Cornell, *University of North Florida*
Kewal Krishan, *Hudson County Community College*
Ferdinand Orock, *Hudson County Community College*
Marnie Phipps, *North Georgia College and State University*
Nancy Schendel, *Iowa Lakes Community College*
Ann Wheeler, *Texas Woman's University*

I would also like to thank the following reviewers, who have given me many useful insights to this and previous editions.

Tony Homayoon Akhlaghi, *Bellevue Community College;* Daniel D. Anderson, *University of Iowa;* Bruce Armbrust, *Lake Tahoe Community College;* Jamie Whitehead Ashby, *Texarkana College;* Teresa Barton, *Western New England College;* Kimberly Bennekin, *Georgia Perimeter College;* Charles M. Biles, *Humboldt State University;* Phyllis Barsch Bolin, *Oklahoma Christian University;* Khristo Boyadzheiv, *Ohio Northern University;* Dave Bregenzer, *Utah State University;* Anne E. Brown, *Indiana University-South Bend;* Diane Burleson, *Central Piedmont Community College;* Beth Burns, *Bowling Green State University;* Alexander Burstein, *University of Rhode Island;* Marilyn Carlson, *University of Kansas;* Victor M. Cornell, *Mesa Community College;* John Dersh, *Grand Rapids Community College;* Jennifer Dollar, *Grand Rapids Community College;* Marcia Drost, *Texas A & M University;* Cameron English, *Rio Hondo College;* Susan E. Enyart, *Otterbein College;* Patricia J. Ernst, *St. Cloud State University;* Eunice Everett, *Seminole Community College;* Kenny Fister, *Murray State University;* Susan C. Fleming, *Virginia Highlands Community College;* Jeff Frost, *Johnson County Community College;* James R. Fryxell, *College of Lake County;* Khadiga H. Gamgoum, *Northern Virginia Community College;* Nicholas E. Geller, *Collin County Community College;* Betty Givan, *Eastern Kentucky University;* Patricia K. Gramling, *Trident Technical College;* Michele Greenfield, *Middlesex County College;* Bernard Greenspan, *University of Akron;* Zenas Hartvigson, *University of Colorado at Denver;* Rodger Hergert, *Rock Valley College;* Allen Hesse, *Rochester Community College;* Rodney Holke-Farnam, *Hawkeye Community College;* Lynda Hollingsworth, *Northwest Missouri State University;* Jean M. Horn, *Northern Virginia Community College;* Spencer Hurd, *The Citadel;* Bill Huston, *Missouri Western State College;* Deborah Johnson, *Cambridge South Dorchester High School;* Francine Winston Johnson, *Howard Community College;* Luella Johnson, *State University of New York, College at Buffalo;* Susan Kellicut, *Seminole Community College;* John Kendall, *Shelby State Community College;* Donna M. Krawczyk, *University of Arizona;* Laura Lake, *Center for Advanced Technologies/ Lakewood High School;* Peter A. Lappan, *Michigan State University;* Charles G. Laws, *Cleveland State Community College;* JoAnn Lewin, *Edison Community College;* Richard J. Maher, *Loyola University;* Carl Main, *Florida College;* Marilyn McCollum, *North Carolina State University;* Judy McInerney, *Sandhills Community College;* David E. Meel, *Bowling Green University;* Beverly Michael, *University of Pittsburgh;* Wendy Morin, *Dwight D. Eisenhower High School;* Roger B. Nelsen, *Lewis and Clark College;* Stephen Nicoloff, *Paradise Valley Community College;* Jon Odell, *Richland Community College;* Paul Oswood, *Ridgewater College;* Wing M. Park, *College of Lake County;* Rupa M. Patel, *University of Portland;* Robert Pearce, *South Plains College;* David R. Peterson, *University of Central Arkansas;* Sandra Poinsett, *College of Southern Maryland;* James Pommersheim, *Reed College;* Antonio Quesada, *University of Akron;* Laura Reger, *Milwaukee Area Technical College;* Jennifer Rhinehart, *Mars Hill College;* Lila F. Roberts, *Georgia Southern University;* Keith Schwingendorf, *Purdue University North Central;* Abdallah Shuaibi, *Truman College;* George W. Shultz, *St. Petersburg Junior College;* Stephen Slack, *Kenyon College;*

Judith Smalling, *St. Petersburg Junior College;* Pamela K. M. Smith, *Fort Lewis College;* Cathryn U. Stark, *Collin County Community College;* Craig M. Steenberg, *Lewis-Clark State College;* Mary Jane Sterling, *Bradley University;* G. Bryan Stewart, *Tarrant County Junior College;* Diane Veneziale, *Burlington County College;* Mahbobeh Vezvaei, *Kent State University;* Ellen Vilas, *York Technical College;* Hayat Weiss, *Middlesex Community College;* Rich West, *Francis Marion University;* Vanessa White, *Southern University;* Howard L. Wilson, *Oregon State University;* Joel E. Wilson, *Eastern Kentucky University;* Michelle Wilson, *Franklin University;* Paul Winterbottom, *Montgomery County Community College;* Fred Worth, *Henderson State University;* Karl M. Zilm, *Lewis and Clark Community College;* Cathleen Zucco-Teveloff, *Rowan University*

I hope that you enjoy learning the mathematics presented in this text. More than that, I hope you gain a new appreciation for the relevance of mathematics to careers in science, technology, business, and medicine.

My thanks to Robert Hostetler, The Behrend College, The Pennsylvania State University, Bruce Edwards, University of Florida, and David Heyd, The Behrend College, The Pennsylvania State University, for their significant contributions to previous editions of this text.

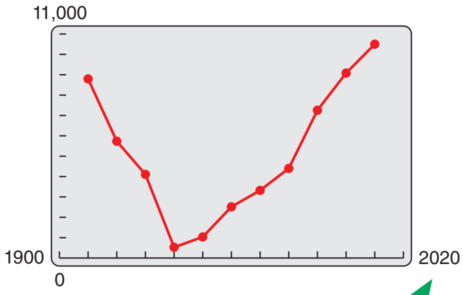
I would also like to thank the staff of Larson Texts, Inc. who assisted in preparing the manuscript, rendering the art package, and typesetting and proofreading the pages and supplements.

On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write me. Over the past two decades I have received many useful comments from both instructors and students, and I value these very much.

Ron Larson, Ph.D.
Professor of Mathematics
Penn State University
www.RonLarson.com

P Prerequisites



Section P.6, Example 5
U.S. Immigrants

P.1 Real Numbers

Real Numbers

Real numbers are used in everyday life to describe quantities such as age, miles per gallon, and population. Real numbers are represented by symbols such as

$$-5, \quad 9, \quad 0, \quad \tfrac{4}{3}, \quad 0.666 \ldots, \quad 28.21, \quad \sqrt{2}, \quad \pi, \quad \text{and} \quad \sqrt[3]{-32}.$$

Here are some important **subsets** (each member of subset B is also a member of set A) of the set of real numbers.

$\{1, 2, 3, 4, \ldots\}$ Set of natural numbers

$\{0, 1, 2, 3, 4, \ldots\}$ Set of whole numbers

$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ Set of integers

A real number is **rational** when it can be written as the ratio p/q of two integers, where $q \neq 0$. For instance, the numbers

$$\frac{1}{3} = 0.3333\ldots = 0.\overline{3}, \quad \frac{1}{8} = 0.125, \quad \text{and} \quad \frac{125}{111} = 1.126126\ldots = 1.\overline{126}$$

are rational. The decimal representation of a rational number either *repeats* $\left(\text{as in } \frac{173}{55} = 3.1\overline{45}\right)$ or *terminates* $\left(\text{as in } \frac{1}{2} = 0.5\right)$. A real number that cannot be written as the ratio of two integers is called **irrational.** Irrational numbers have infinite nonrepeating decimal representations. For instance, the numbers

$$\sqrt{2} = 1.4142135\ldots \approx 1.41$$

and

$$\pi = 3.1415926\ldots \approx 3.14$$

are irrational. (The symbol $\approx$ means "is approximately equal to.") Figure P.1 shows subsets of real numbers and their relationships to each other.

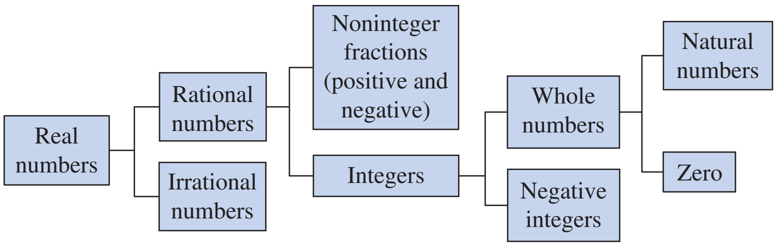


Figure P.1 *Subsets of Real Numbers*

Real numbers are represented graphically by a **real number line.** The point 0 on the real number line is the **origin.** Numbers to the right of 0 are positive and numbers to the left of 0 are negative, as shown in Figure P.2. The term **nonnegative** describes a number that is either positive or zero.

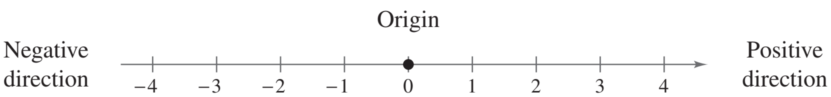


Figure P.2 *The Real Number Line*

There is a *one-to-one correspondence* between real numbers and points on the real number line. That is, every point on the real number line corresponds to exactly one real number, called its **coordinate,** and every real number corresponds to exactly one point on the real number line, as shown in Figure P.3.

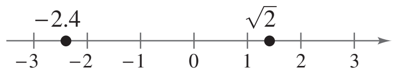
What you should learn

- Represent and classify real numbers.
- Order real numbers and use inequalities.
- Find the absolute values of real numbers and the distance between two real numbers.
- Evaluate algebraic expressions and use the basic rules and properties of algebra.

Why you should learn it

Real numbers are used in every aspect of our lives, such as finding the surplus or deficit in the federal budget. See Exercises 89–94 on page 10.

Every point on the real number line corresponds to exactly one real number.



Every real number corresponds to exactly one point on the real number line.

Figure P.3 *One-to-One Correspondence*

Ordering Real Numbers

One important property of real numbers is that they are **ordered.**

Definition of Order on the Real Number Line

If a and b are real numbers, then a is **less than** b when $b - a$ is positive. This order is denoted by the **inequality** $a < b$. This relationship can also be described by saying that b is **greater than** a and writing $b > a$. The inequality $a \leq b$ means that a is **less than or equal to** b, and the inequality $b \geq a$ means that b is **greater than or equal to** a. The symbols $<$, $>$, $\leq$, and $\geq$ are **inequality symbols.**

Geometrically, this definition implies that $a < b$ if and only if a lies to the *left* of b on the real number line, as shown in Figure P.4.

Figure P.4 *$a < b$ if and only if a lies to the left of b.*

EXAMPLE 1 Interpreting Inequalities

See LarsonPrecalculus.com for an interactive version of this type of example.

Describe the subset of real numbers that the inequality represents.

a. $x \leq 2$ **b.** $x > -1$ **c.** $-2 \leq x < 3$

Solution

a. The inequality $x \leq 2$ denotes all real numbers less than or equal to 2, as shown in Figure P.5.

b. The inequality $x > -1$ denotes all real numbers greater than -1, as shown in Figure P.6.

c. The inequality $-2 \leq x < 3$ means that $x \geq -2$ *and* $x < 3$. The "double inequality" denotes all real numbers between -2 and 3, including -2 but not including 3, as shown in Figure P.7.

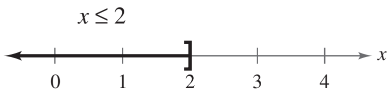


Figure P.5

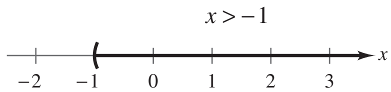


Figure P.6

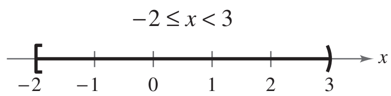


Figure P.7

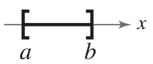
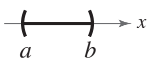
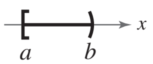
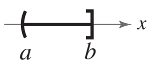
✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Describe the subset of real numbers that the inequality represents.

a. $x > -3$ **b.** $0 < x \leq 4$

Inequalities can be used to describe subsets of real numbers called **intervals.** In the bounded intervals below, the real numbers a and b are the **endpoints** of each interval.

Bounded Intervals on the Real Number Line

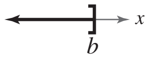
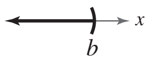
Notation	*Interval Type*	*Inequality*	*Graph*
$[a, b]$	Closed	$a \leq x \leq b$	
(a, b)	Open	$a < x < b$	
$[a, b)$		$a \leq x < b$	
$(a, b]$		$a < x \leq b$	

Remark

The endpoints of a closed interval are included in the interval. The endpoints of an open interval are *not* included in the interval.

The symbols ∞, **positive infinity,** and $-\infty$, **negative infinity,** do not represent real numbers. They are simply convenient symbols used to describe the unboundedness of an interval, such as $(1, \infty)$ or $(-\infty, 3]$.

Unbounded Intervals on the Real Number Line

Notation	*Interval Type*	*Inequality*	*Graph*
$[a, \infty)$		$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$		$x \leq b$	
$(-\infty, b)$	Open	$x < b$	
$(-\infty, \infty)$	Entire real line	$-\infty < x < \infty$	

Remark

An interval is unbounded when it continues indefinitely in one or both directions.

EXAMPLE 2 Using Inequalities to Represent Intervals

Use inequality notation to represent each of the following.

a. c is at most 2.

b. All x in the interval $(-3, 5]$

c. t is at least 4 but less than 11.

Solution

a. The statement "c is at most 2" can be represented by $c \leq 2$.

b. "All x in the interval $(-3, 5]$" can be represented by $-3 < x \leq 5$.

c. The statement "t is at least 4 but less than 11" can be represented by $4 \leq t < 11$.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Use inequality notation to represent the statement "x is greater than -2 and at most 4."

EXAMPLE 3 Interpreting Intervals

Give a verbal description of each interval.

a. $(-1, 0)$

b. $[2, \infty)$

c. $(-\infty, 0)$

Solution

a. This interval consists of all real numbers that are greater than -1 and less than 0.

b. This interval consists of all real numbers that are greater than or equal to 2.

c. This interval consists of all negative real numbers.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Give a verbal description of the interval $[-2, 5)$.

The **Law of Trichotomy** states that for any two real numbers a and b, precisely one of three relationships is possible:

$$a = b, \quad a < b, \quad \text{or} \quad a > b. \qquad \text{Law of Trichotomy}$$

Absolute Value and Distance

The **absolute value** of a real number is its *magnitude*, or the distance between the origin and the point representing the real number on the real number line.

Definition of Absolute Value

If a is a real number, then the **absolute value** of a is

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}.$$

Notice from this definition that the absolute value of a real number is never negative. For instance, if $a = -5$, then $|-5| = -(-5) = 5$. The absolute value of a real number is either positive or zero. Moreover, 0 is the only real number whose absolute value is 0. So, $|0| = 0$.

Explore the Concept

Absolute value expressions can be evaluated on a graphing utility. When evaluating an expression such as $|3 - 8|$, parentheses should surround the expression, as shown below. Evaluate each expression. What can you conclude?

a. $|6|$ **b.** $|-1|$

c. $|5 - 2|$ **d.** $|2 - 5|$

EXAMPLE 4 Evaluating an Absolute Value Expression

Evaluate $\dfrac{|x|}{x}$ for (a) $x > 0$ and (b) $x < 0$.

Solution

a. If $x > 0$, then $|x| = x$ and $\dfrac{|x|}{x} = \dfrac{x}{x} = 1$.

b. If $x < 0$, then $|x| = -x$ and $\dfrac{|x|}{x} = \dfrac{-x}{x} = -1$.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Evaluate $\dfrac{|x + 3|}{x + 3}$ for (a) $x > -3$ and (b) $x < -3$.

Properties of Absolute Value

1. $|a| \geq 0$ **2.** $|-a| = |a|$

3. $|ab| = |a||b|$ **4.** $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \quad b \neq 0$

Absolute value can be used to define the distance between two points on the real number line. For instance, the distance between -3 and 4 is

$$|-3 - 4| = |-7| = 7$$

as shown in Figure P.8.

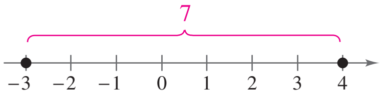


Figure P.8 *The distance between −3 and 4 is 7.*

Distance Between Two Points on the Real Number Line

Let a and b be real numbers. The **distance between a and b** is

$$d(a, b) = |b - a| = |a - b|.$$

Algebraic Expressions and the Basic Rules of Algebra

One characteristic of algebra is the use of letters to represent numbers. The letters are **variables,** and combinations of letters and numbers are **algebraic expressions.** Here are a few examples of algebraic expressions.

$$5x, \quad 2x - 3, \quad \frac{4}{x^2 + 2}, \quad 7x + y$$

Definition of an Algebraic Expression

An **algebraic expression** is a combination of letters (**variables**) and real numbers (**constants**) combined using the operations of addition, subtraction, multiplication, division, and exponentiation.

The **terms** of an algebraic expression are those parts that are separated by *addition*. For example,

$$x^2 - 5x + 8 = x^2 + (-5x) + 8$$

has three terms: x^2 and $-5x$ are the **variable terms,** and 8 is the **constant term.** The numerical factor of a term is called the **coefficient.** For instance, the coefficient of $-5x$ is -5, and the coefficient of x^2 is 1.

To **evaluate** an algebraic expression, substitute numerical values for each of the variables in the expression.

EXAMPLE 5 Evaluating Algebraic Expressions

	Expression	*Value of Variable*	*Substitute*	*Value of Expression*
a.	$-3x + 5$	$x = 3$	$-3(3) + 5$	$-9 + 5 = -4$
b.	$3x^2 + 2x - 1$	$x = -1$	$3(-1)^2 + 2(-1) - 1$	$3 - 2 - 1 = 0$
c.	$\frac{2x}{x + 1}$	$x = -3$	$\frac{2(-3)}{-3 + 1}$	$\frac{-6}{-2} = 3$

Note that you must substitute the value for *each* occurrence of the variable.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Evaluate $4x - 5$ when $x = 0$.

When an algebraic expression is evaluated, the **Substitution Principle** is used. It states, "If $a = b$, then a can be replaced by b in any expression involving a." For instance, in Example 5(a), 3 is substituted for x in the expression $-3x + 5$.

There are four arithmetic operations with real numbers: addition, multiplication, subtraction, and division, denoted by the symbols

$$+, \quad \times \text{ or } \cdot, \quad -, \quad \text{and} \quad \div \text{ or } /.$$

Of these, addition and multiplication are the two primary operations. Subtraction and division are the inverse operations of addition and multiplication, respectively.

Subtraction: Add the opposite of b. *Division: Multiply by the reciprocal of b.*

$$a - b = a + (-b)$$

$$\text{If } b \neq 0, \text{ then } a/b = a\left(\frac{1}{b}\right) = \frac{a}{b}.$$

In these definitions, $-b$ is the **additive inverse** (or opposite) of b, and $1/b$ is the **multiplicative inverse** (or reciprocal) of b. In the fractional form a/b, a is the **numerator** of the fraction and b is the **denominator.**

Because the properties of real numbers below are true for variables and algebraic expressions, as well as for real numbers, they are often called the **Basic Rules of Algebra.** Try to formulate a verbal description of each property. For instance, the Commutative Property of Addition states that *the order in which two real numbers are added does not affect their sum.*

Basic Rules of Algebra

Let a, b, and c be real numbers, variables, or algebraic expressions.

	Property	*Example*
Commutative Property of Addition:	$a + b = b + a$	$4x + x^2 = x^2 + 4x$
Commutative Property of Multiplication:	$ab = ba$	$(1 - x)x^2 = x^2(1 - x)$
Associative Property of Addition:	$(a + b) + c = a + (b + c)$	$(x + 5) + x^2 = x + (5 + x^2)$
Associative Property of Multiplication:	$(ab)c = a(bc)$	$(2x \cdot 3y)(8) = (2x)(3y \cdot 8)$
Distributive Properties:	$a(b + c) = ab + ac$	$3x(5 + 2x) = 3x \cdot 5 + 3x \cdot 2x$
	$(a + b)c = ac + bc$	$(y + 8)y = y \cdot y + 8 \cdot y$
Additive Identity Property:	$a + 0 = a$	$5y^2 + 0 = 5y^2$
Multiplicative Identity Property:	$a \cdot 1 = a$	$(4x^2)(1) = 4x^2$
Additive Inverse Property:	$a + (-a) = 0$	$6x^3 + (-6x^3) = 0$
Multiplicative Inverse Property:	$a \cdot \frac{1}{a} = 1, \ a \neq 0$	$(x^2 + 4)\left(\frac{1}{x^2 + 4}\right) = 1$

Because subtraction is defined as "adding the opposite," the Distributive Properties are also true for subtraction. For instance, the "subtraction form" of $a(b + c) = ab + ac$ is written as

$$a(b - c) = ab - ac.$$

Properties of Negation and Equality

Let a, b, and c be real numbers, variables, or algebraic expressions.

Property	*Example*
1. $(-1)a = -a$	$(-1)7 = -7$
2. $-(-a) = a$	$-(-6) = 6$
3. $(-a)b = -(ab) = a(-b)$	$(-5)3 = -(5 \cdot 3) = 5(-3)$
4. $(-a)(-b) = ab$	$(-2)(-x) = 2x$
5. $-(a + b) = (-a) + (-b)$	$-(x + 8) = (-x) + (-8) = -x - 8$
6. If $a = b$, then $a + c = b + c$.	$\frac{1}{2} + 3 = 0.5 + 3$
7. If $a = b$, then $ac = bc$.	$4^2(2) = 16(2)$
8. If $a \pm c = b \pm c$, then $a = b$.	$1.4 - 1 = \frac{7}{5} - 1 \implies 1.4 = \frac{7}{5}$
9. If $ac = bc$ and $c \neq 0$, then $a = b$.	$3x = 3 \cdot 4 \implies x = 4$

Remark

Be sure you see the difference between the *opposite of a number* and a *negative number.* If a is already negative, then its opposite, $-a$, is positive. For instance, if $a = -2$, then $-a = -(-2) = 2$.

Properties of Zero

Let a and b be real numbers, variables, or algebraic expressions.

1. $a + 0 = a$ and $a - 0 = a$

2. $a \cdot 0 = 0$

3. $\dfrac{0}{a} = 0, \quad a \neq 0$

4. $\dfrac{a}{0}$ is undefined.

5. Zero-Factor Property: If $ab = 0$, then $a = 0$ or $b = 0$.

The "or" in the Zero-Factor Property includes the possibility that either or both factors may be zero. This is an **inclusive or,** and it is the way the word "or" is generally used in mathematics.

Properties and Operations of Fractions

Let a, b, c, and d be real numbers, variables, or algebraic expressions such that $b \neq 0$ and $d \neq 0$.

1. Equivalent Fractions: $\dfrac{a}{b} = \dfrac{c}{d}$ if and only if $ad = bc$.

2. Rules of Signs: $-\dfrac{a}{b} = \dfrac{-a}{b} = \dfrac{a}{-b}$ and $\dfrac{-a}{-b} = \dfrac{a}{b}$

3. Generate Equivalent Fractions: $\dfrac{a}{b} = \dfrac{ac}{bc}, \quad c \neq 0$

4. Add or Subtract with Like Denominators: $\dfrac{a}{b} \pm \dfrac{c}{b} = \dfrac{a \pm c}{b}$

5. Add or Subtract with Unlike Denominators: $\dfrac{a}{b} \pm \dfrac{c}{d} = \dfrac{ad \pm bc}{bd}$

6. Multiply Fractions: $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$

7. Divide Fractions: $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}, \quad c \neq 0$

Remark

In Property 1, the phrase "if and only if" implies two statements. One statement is: If $a/b = c/d$, then $ad = bc$. The other statement is: If $ad = bc$, where $b \neq 0$ and $d \neq 0$, then $a/b = c/d$.

EXAMPLE 6 Properties and Operations of Fractions

a. Equivalent fractions: $\dfrac{x}{5} = \dfrac{3 \cdot x}{3 \cdot 5} = \dfrac{3x}{15}$

b. Divide fractions: $\dfrac{7}{x} \div \dfrac{3}{2} = \dfrac{7}{x} \cdot \dfrac{2}{3} = \dfrac{14}{3x}$

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com.

a. Multiply fractions: $\dfrac{3}{5} \cdot \dfrac{x}{6}$

b. Add fractions: $\dfrac{x}{10} + \dfrac{2x}{5}$

If a, b, and c are integers such that $ab = c$, then a and b are **factors** or **divisors** −41
of c. A **prime number** is an integer that has exactly two positive factors: itself and 1. For example, 2, 3, 5, 7, and 11 are prime numbers. The numbers 4, 6, 8, 9, and 10 are **composite** because they can be written as the product of two or more prime numbers. The number 1 is neither prime nor composite. The **Fundamental Theorem of Arithmetic** states that every positive integer greater than 1 can be written as the product of prime numbers. For instance, the **prime factorization** of 24 is

$$24 = 2 \cdot 2 \cdot 2 \cdot 3.$$

P.1 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.
For instructions on how to use a graphing utility, see Appendix A.

Vocabulary and Concept Check

In Exercises 1–5, fill in the blank(s).

1. A real number is _______ when it can be written as the ratio $\frac{p}{q}$ of two integers, where $q \neq 0$.

2. _______ numbers have infinite nonrepeating decimal representations.

3. A _______ number is an integer with exactly two positive factors: itself and 1.

4. An algebraic expression is a combination of letters called _______ and real numbers called _______ .

5. The _______ of an algebraic expression are those parts separated by addition.

6. Is $|5 - 2| = |2 - 5|$?

In Exercises 7–12, match each property with its name.

7. Commutative Property of Addition	(a) $a \cdot 1 = a$
8. Associative Property of Multiplication	(b) $a(b + c) = ab + ac$
9. Additive Inverse Property	(c) $a + b = b + a$
10. Distributive Property	(d) $(ab)c = a(bc)$
11. Associative Property of Addition	(e) $a + (-a) = 0$
12. Multiplicative Identity Property	(f) $(a + b) + c = a + (b + c)$

Procedures and Problem Solving

Identifying Subsets of Real Numbers **In Exercises 13–18, determine which numbers are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.**

13. $\left\{-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{2}, 0, 1, -4, -1\right\}$

14. $\left\{\sqrt{5}, -7, -\frac{7}{3}, 0, 3.12, \frac{5}{4}, -2, -8, 3\right\}$

15. $\{2.01, 0.666\ldots, -13, 0.010110111\ldots, 1, -10, 20\}$

16. $\left\{2.3030030003\ldots, 0.7575, -4.63, \sqrt{10}, -2, 0.3, 8\right\}$

17. $\left\{-\pi, -\frac{1}{3}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -2, 3, -3\right\}$

18. $\left\{25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 6, -4, 18\right\}$

Finding the Decimal Form of a Rational Number **In Exercises 19–24, use a calculator to find the decimal form of the rational number. If it is a nonterminating decimal, write the repeating pattern.**

19. $\frac{5}{16}$ **20.** $\frac{17}{4}$

21. $\frac{41}{333}$ **22.** $\frac{3}{7}$

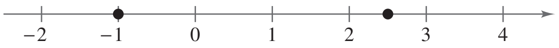
23. $-\frac{100}{11}$ **24.** $-\frac{218}{33}$

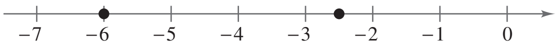
Writing a Decimal as a Fraction **In Exercises 25–28, use a calculator to rewrite the rational number as the ratio of two integers.**

25. 6.4 **26.** -7.5

27. -12.3 **28.** 1.87

Writing an Inequality **In Exercises 29 and 30, approximate the numbers and place the correct inequality symbol (< or >) between them.**

29.


30.


Plotting Real Numbers **In Exercises 31–36, plot the two real numbers on the real number line. Then place the correct inequality symbol (< or >) between them.**

31. $-4, 2$ **32.** $-3.5, 1$

33. $\frac{3}{2}, -\frac{7}{2}$ **34.** $-\frac{8}{7}, -\frac{3}{7}$

35. $-\frac{3}{4}, -\frac{5}{8}$ **36.** $\frac{5}{6}, \frac{2}{3}$

Interpreting Inequalities **In Exercises 37–44, (a) verbally describe the subset of real numbers represented by the inequality, (b) sketch the subset on the real number line, and (c) state whether the interval is bounded or unbounded.**

37. $x \leq 5$ **38.** $x > -3$

39. $x < 0$ **40.** $x \geq 4$

41. $-2 < x < 2$

42. $0 \leq x \leq 5$

43. $-1 \leq x < 0$

44. $-9 < x \leq -6$